Int J Theor Phys (2010) 49: 2476–2485
DOI 10.1007/s10773-010-0433-z

# Planck Oscillators in the Background Dark Energy

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Received: 10 April 2010 / Accepted: 14 July 2010 / Published online: 3 August 2010


**Abstract** We consider a model for an underpinning of the universe: there are oscillators at the Planck scale in the background dark energy. Starting from a coherent array of such oscillators it is possible to get a description from elementary particles to Black Holes including the usual Hawking-Beckenstein theory. There is also a description of Gravitation in the above model which points to a unified description with electromagnetism.

**Keywords** Planck · Oscillators · Dark · Energy

## 1 Introduction

Max Planck, more than a century ago introduced a combination of the well known fundamental constants, $\hbar, G, c$ that gave a length, mass and time scale viz.,

$$l = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-33} \text{ cm},$$
$$m = \sqrt{\frac{\hbar c}{G}} \sim 10^{-5} \text{ g}, \tag{1}$$
$$t = \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-42} \text{ s}.$$

Based on the Paper at the Max Born Symposium, 2009, Wroclaw.

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We can easily verify that $l$ plays the role of the Compton length and the Schwarzchild radius of a black hole of the mass $m$ [1]

$$l = \frac{\hbar}{2mc}, \qquad l = \frac{2Gm}{c^2}. \tag{2}$$

Today in various Quantum Gravity approaches including String theory, the Planck length $l$ is considered to be the fundamental minimum length, and so also the time interval $t$.

## 2 Quantum Strings

In spite of great success, the standard theory has failed to quantize gravitation. One of the obstacles has been the point spacetime concept ingrained in these theories leading to infinities. For the past few decades Quantum Gravity schemes as also String theory have attempted to break out of this limitation. Let us first consider string theory.

Regge's work of the fifties [2–4] suggested that resonances had angular momentum, on the one hand and resembled extended objects, that is particles smeared out in space.

Then, in 1968, G. Veneziano came up with a unified description of the Regge resonances and other scattering processes. Veneziano considered the collision and scattering process as a black box and pointed out that there were in essence, two scattering channels, $s$ and $t$ channels. These, he argued gave a dual description of the same process [5, 6].

There is another interesting hint which we get from Quantum Chromo Dynamics. Let us come to the inter-quark potential [7, 8]. There are two interesting features of this potential. The first is that of confinement, which is given by a potential term like

$$V(r) \approx \sigma r, \quad r \to \infty,$$

where $\sigma$ is a constant. This describes the large distance behavior between two quarks. The confining potential ensures that quarks do not break out of their bound state, which means that effectively free quarks cannot be observed.

The second interesting feature is asymptotic freedom. This is realized by a Coulumbic potential

$$V_c(r) \approx -\frac{\alpha(r)}{r} \quad (\text{small } r) \quad \text{where } \alpha(r) \sim \frac{1}{\ln(1/\lambda^2 r^2)}.$$

The constant $\sigma$ is called the string tension, because there are string models which yield $V(r)$. This is because, at large distances the inter-quark field is string like with the energy content per unit length becoming constant. Use of the angular momentum–mass relation indicates that $\sigma \sim (400\ \text{MeV})^2$.

Such considerations lead to strings which are governed by the equation [9–12]

$$\rho\ddot{y} - Ty'' = 0, \tag{3}$$

$$\omega = \frac{\pi}{2l}\sqrt{\frac{T}{\rho}}, \tag{4}$$

$$T = \frac{mc^2}{l}; \qquad \rho = \frac{m}{l}, \tag{5}$$

$$\sqrt{T/\rho} = c, \tag{6}$$

$T$ being the tension of the string, $l$ its length and $\rho$ the line density and $\omega$ in (4) the frequency. The identification (4), (5) gives (6), where $c$ is the velocity of light, and (1) then goes over to the usual d'Alembertian or massless Klein-Gordon equation. (It is worth noting that as $l \to 0$ the potential energy which is $\sim \int_0^l T(\partial y/\partial x)^2 dx$ rapidly oscillates.)

Further, if the above string is quantized canonically, we get

$$\langle \Delta x^2 \rangle \sim l^2. \tag{7}$$

The string effectively shows up as an infinite collection of harmonic oscillators [10]. It must be mentioned that (7) and (4) to (6) both show that $l$ is of the order of the Compton wavelength. This has been called one of the miracles of string theory by Veneziano [13]. In fact the minimum length $l$ turns out to be given by $T/\hbar^2 = c/l^2$, which from (5) and (6) is seen to give the Compton wavelength.

This is a description of what may be called a "Bosonic String". These theories have certain technical problems, for example they allow the existence of tachyons. Further they do not easily meet the requirements of Quantum theory, as for example the commutation relations. The difficulties are resolved only in twenty six dimensions.

If the relativistic quantized string is given rotation [14], then we get back the equation for the Regge trajectories. Here we are dealing with objects of finite extension rotating with the velocity of light rather like spinning black holes. It must be pointed out that, in superstring theory, there is an additional term $a_0$

$$J \leq (2\pi T)^{-1} M^2 + a_0 \hbar, \quad \text{with } a_0 = +1(+2) \text{ for the open (closed) string.} \tag{8}$$

In (8) $a_0$ comes from a zero-point energy effect. When $a_0 = 1$ we have the usual gauge bosons and when $a_0 = 2$ we have the gravitons.

The theory of Quantum Super Strings in contrast requires only ten dimensions. Here, Quantum operators describing anti-commuting variables satisfy anti-commutation relations. Indeed this bivalence is a hallmark of supersymmetry itself.

The extra dimensions that appear in String theories reduce to the four dimensions of the physical spacetime by virtue of the fact that the redundant dimensions are treated as curled up into a negligible extension, in the manner suggested by Kaluza and later Klein in the early twentieth century. Kaluza's original motivation had been to unify electromagnetism and gravitation by introducing a fifth negligible coordinate. The curling up takes place at the Planck scale [15].

The interesting thing about Quantum Superstring theory is the natural emergence of the spin 2 graviton as can be seen from (8), or as Witten puts it, the theory "predicts" gravitation.

Meanwhile Supersymmetry or SUSY developed in parallel. This theory requires that each particle with integral spin has a counterpart with the same mass but having half integral spin. That is Bosons have their supersymmetric counterparts in Fermions. SUSY is then broken so that the counterparts would have a much greater mass, which would then account for the fact that these latter have not been observed. Nevertheless the fact that in this theory gravitation can be unified with the other forces makes it attractive.

So in the mid eighties ten dimensional QSS displaced Supergravity. There were five QSS theories—$E_8 \times E_8$ heterotic, $SO(32)$ heterotic, the Type I, the Type IIA and Type IIB. Of these the Type I is an open string while the others form closed loops. The $E_8 \times E_8$ appeared to explain many features of elementary particles and their forces.

However there were some disturbing questions. Why were there five different theories? After all we need a unique theory. And then why ten dimensions, while Supersymmetry allows eleven dimensions? Another not very convincing factor was the fact that particles were being represented as one dimensional strings. Surely a more general formulation would have two dimensional surfaces or membranes or even p-dimensional entities which we may call p-branes. This generalization resembles the earlier attempt of Dirac's representing particles as a shell or membrane. Infact if the radius of the circle shrinks, the membrane begins to resemble a rolled up object in ten dimensions. It reduces to a Type IIA Superstring.

In such deformations certain topological properties can remain conserved. Over the past few years, a variant called $M$ Theory arising from these generalizations has attracted much attention. This theory also uses Supersymmetry, which is broken so that the postulated particles do not have the same mass as the known particles. Under SUSY there can be a maximum of eleven dimensions, the extra dimensions being curled up as in Kaluza-Klein theories.

The eleventh and extra dimension of the M-Theory could be shrunk, so that there would be two ten dimensional universes connected by the eleven dimensional spacetime. Now particles and strings would exist in the parallel universes which can interact through gravitation. The interesting aspect of the above scenario is that it is possible to conceive of all the four interactions converging at an energy far less than the Planck energy ($10^{19}$ GeV). Infact the Planck energy is so high that it is beyond forseeable experiments. Thus this would bring the eleven dimensional M-Theory closer to experiment. There have been further developments involving what are called Dirichlet surfaces. It is now suspected that black holes can be treated as intersecting black branes wrapped around seven curled up dimensions. There is here, an interesting interface between M-Theory and black hole physics [16]. In M-Theory, the position coordinates become matrices and this leads to, a noncommutative geometry or fuzzy spacetime in which spacetime points are no longer well defined [17]

$$[x, y] \neq 0.$$

From this point of view the mysterious $M$ in M-Theory could stand for Matrix, rather than Membrane.

So M-Theory is the new avatar of QSS. Nevertheless it is still far from being the last word. There are still any number of routes for compressing ten dimensions to our four dimensions. There is still no contact with experiment. It also appears that these theories lead to an unacceptably high cosmological constant and so on.

To bypass these difficulties, string theorists have had to invoke the concept of a landscape of universes together with the anthropic principle. The idea is that each of the $100^{500}$ or so solutions represents a universe, each with its own characteristic values for physical constants. The anthropic principle is then invoked to explain why our universe has the observed values for the physical constant, and this includes the cosmological constant. All this however has not gone well with many physicists and the entire spectrum of string theory has come under severe criticism in the past few years [18–20]. Even prominent string theorists like David Gross now express pessimism about string theory. As Susskind puts it, [21] "Confusion and disorientation reign; cause and effect break down; certainty evaporates; all the old rules fail. That's what happens when the dominant paradigm breaks down." Let us explore further, given the above context.

## 3 The Planck Oscillators

Spacetime intervals smaller than given in (1) and (2) are meaningless both classically and Quantum mechanically. Classically because we cannot penetrate the Schwarzchild radius, and Quantum mechanically because we encounter unphysical phenomena inside a typical Compton scale. All this has been discussed in greater detail in the literature (cf. Ref. [22] and several references therein). We could of course go to smaller intervals by abandoning the Planck mass and the fundamental constants in (1) and (2)—we will come back to this point a little later. In any case, it is worth pointing out that Quantum mechanically it is meaningless to speak about spacetime points, as these would imply infinite momenta and energy.

At another level, it may be mentioned that the author's 1997 model invoked a background dark energy and fluctuations therein to deduce a model of the universe that was accelerating with a small cosmological constant, together with several other relations completely consistent with Astrophysics and Cosmology (cf. Ref. [23] and several references therein).

It is against this backdrop and the difficulties with Quantum Gravity approaches as detailed in Sect. 2, that the author had put forward his model of Planck oscillators in the dark energy driven Quantum vacuum, several years ago (cf. Ref. [24] and several references therein, [25] and [34]). To illustrate this model let us consider an array of $N$ particles, spaced a distance $\Delta x$ apart, which behave like oscillators that are connected by springs. As is known we then have [24, 26–28] (cf. in particular Ref. [28])

$$\begin{aligned} r &= \sqrt{N\Delta x^2}, \\ ka^2 &\equiv k\Delta x^2 = \frac{1}{2}k_B T \end{aligned} \tag{9}$$

where $k_B$ is the Boltzmann constant, $T$ the temperature, $r$ the total extension and $k$ is the spring constant given by

$$\omega_0^2 = \frac{k}{m}, \tag{10}$$

$$\omega = \left(\frac{k}{m}a^2\right)^{\frac{1}{2}}\frac{1}{r} = \omega_0\frac{a}{r}. \tag{11}$$

It must be pointed out that (9) to (11) are quite general and a part of the well known theory referred to in [26–28]. In particular there is no restriction on the temperature $T$. $m$ and $\omega$ are the mass of the particle and frequency of oscillation. In (10) $\omega_0$ is the frequency of the individual oscillator, while in (11) $\omega$ is the frequency of the array of $N$ oscillators, $N$ given in (9).

We now take the mass of the particles to be the Planck mass and set $\Delta x \equiv a = l$, the Planck length as the mass and length are free parameters. In other words, instead of considering a single Planck oscillator as in String theory, we are now considering a coherent (coupled) array of such oscillators, rather like coherent vibrating atoms in a linear crystal. We also use the well known Einstein-de Broglie relations that give quite generally the frequency in terms of energy and mass.

$$E = \hbar\omega = mc^2. \tag{12}$$

It may be immediately observed that if we use (10) and (9) we can deduce that

$$k_B T \sim mc^2.$$

Independently of the above steps this agrees with the (Beckenstein) temperature of a Black Hole of Planck mass in the usual theory. Indeed as noted, Rosen [29] had shown that a

Planck mass particle at the Planck scale can be considered to be a Universe in itself with a Schwarzchild radius equalling the Planck length.

Thus we have shown from the above theory of oscillators that an oscillator with the Planck mass and with a spatial extent at the Planck scale has the same temperature as the Beckenstein temperature of a Schwarzchild Black Hole of mass given by the Planck mass. The above results can also be obtained by a different route as described in [30].

## 4 Elementary Particles and Black Hole Thermodynamics

We have also argued elsewhere that, given the well known effect that the universe consists of $N \sim 10^{80}$ elementary particles like the pion, it is possible to deduce that a typical elementary particle consists of $n \sim 10^{40}$ Planck oscillators. As this has been discussed extensively in the references given, we merely quote the final result. These form a coherent array of $n$ elements described by (9) to (12) above. In this case $N$ in (9) becomes $n$ and we can immediately deduce the following

$$l_\pi = \sqrt{n} l, \qquad m_\pi = \frac{m}{\sqrt{n}} \tag{13}$$

which give the Compton wavelength and mass of a typical elementary particle represented by $l_\pi$ and $m_\pi$. So a typical elementary particle is given as the lowest energy state of the above coherent array of $n$ Planck oscillators.

Interestingly the above description can lead to an immediate correspondence with black hole thermodynamics. We now rewrite (11) as, (interchanging the roles of $\omega$ and $\omega_0$),

$$\omega_0 = \frac{r}{a}\omega.$$

Remembering that, quite generally, the frequency and mass are related by (12), i.e.,

$$\omega = \frac{mc^2}{\hbar},$$

we get on using (9)

$$\hbar\omega\left\langle \frac{l}{r} \right\rangle^{-1} \approx mc^2 \times \frac{r}{l} \approx Mc^2 = \sqrt{\bar{N}} mc^2 \tag{14}$$

where we now consider not the lowest energy states of the array as previously but rather energy states much higher than the Planck energy. Generally, if an arbitrary mass $M$, as in (14), is given in terms of $\bar{N}$ Planck oscillators, in the above model, then we have from (14) and (9):

$$M = \sqrt{\bar{N}} m \quad \text{and} \quad R = \sqrt{\bar{N}} l, \tag{15}$$

where $R$ is the radius or extension of the object. We must stress the factor $\sqrt{\bar{N}}$ in (15), arising from the fact that the oscillators are coupled, as given in (9). If the oscillators had not been coupled, or equivalently had not formed a coherent system, then we would have, for example, $M = \bar{N} m$ or $R = \bar{N} l$ instead of (15). Using the fact that $l$ has been chosen to be the Schwarzchild radius of the (Planck) mass $m$, this gives immediately,

$$R = 2GM/c^2$$

This shows that if an arbitrary mass $M$ consists of $\bar{N}$ coherent Planck oscillators as above, and specifically (15), then its radius $R$ is given by the above expression, which is its Schwarzchild radius. In other words, such an object shows up as a Schwarzchild Black Hole. It must be emphasized that the expression for $R$ follows from the theory of oscillators, specifically (15) and shows that it is identical to the Schwarzchild radius for the same mass $M$. We have merely used the known equivalence of the Planck length and Schwarzchild radius for the Planck mass.

## 5 Thermodynamic Gravitation

We can push the above consideration further. So far we have considered only a coherent array [31]. This is necessary for meaningful physics and leads to the elementary particle masses and their other parameters as seen above. Cercignani [32] had used Quantum oscillations, though just before the dark energy era—these were the usual Zero Point oscillations, which had also been invoked by the author in his model. Invoking gravitation, what he proved was, in his own words, "Because of the equivalence of mass and energy, we can estimate that this (i.e. chaotic oscillations) will occur when the former will be of the order of $G[(\hbar\omega)c^{-2}]^2[\omega^{-1}c]^{-1} = G\hbar^2\omega^3c^{-5}$, where $G$ is the constant of gravitational attraction and we have used as distance the wavelength. This must be less than the typical electromagnetic energy $\hbar\omega$. Hence $\omega$ must be less than $(G\hbar)^{-1/2}c^{5/2}$, which gives a gravitational cut off for the frequency in the zero-point energy."

In other words he deduced that there has to be a maximum cut off frequency of oscillators given by

$$G\hbar\omega_{max}^2 = c^5 \tag{16}$$

for the very existence of coherent oscillations. We would like to point out that if we use the well known equation encountered above namely

$$\hbar\omega = mc^2,$$

in (16) we get the well known relation

$$Gm_P^2 \approx \hbar c \tag{17}$$

which shows that at the Planck scale the gravitational and electromagnetic strengths are of the same order. This is not surprising because it was the very basis of Cercignani's derivation—if indeed the gravitational energy is greater than that given in (17) that is greater than the electromagnetic energy, then the Zero Point oscillators, which we have called the Planck oscillators would become chaotic and incoherent—there would be no physics.

Let us now speak only in terms of the background dark energy. Then we can argue that (17) is the necessary and sufficient condition for coherent Planck oscillators to exist, in order that there be elementary particles as given by (13) and the rest of the requirements for the meaningful physical universe. In other words gravitational energy represented by the gravitation constant $G$ given in (17) is a measure of the energy from the Quantum background that allows a physically meaningful universe—in this sense it is not a separate fundamental interaction. We will return to this point.

It is interesting that (17) also arises in Sakharov's treatment of gravitation where it is a residual type of a zero point energy [22, 33].

To proceed if we use (13) in (17) we can easily deduce

$$Gm^2 \approx \frac{e^2}{n} = \frac{e^2}{\sqrt{N}} \tag{18}$$

where now $N \sim 10^{80}$, the number of particles in the universe.

Equation (18) has been known for a long time empirically, as an accident without any fundamental explanation. Here we have deduced it on the basis of the Planck oscillator model. Equation (18) too brings out the relation between gravitation and the background Zero Point Field or Quantum vacuum or dark energy. It shows that the gravitational energy has the same origin as the electromagnetic energy but is in a sense a smeared out effect over the $N$ particles of the universe. We will argue in Sect. 7 that the smearing out is due to the fact that we require an array of oscillators. In the context of the above considerations we can now even claim that (18) gives the desired unified description of electromagnetism and gravitation and not an ad hoc formula.

## 6 Black Holes Again

If we use (10), (9) and (16) we get [34]

$$k_B T = m\omega_{max}^2 l^2 = \frac{c^5/G\hbar}{ml^2} = \frac{\hbar c^3}{Gm},$$

remembering that $l$ by (2) is also the Compton wavelength. That is we get

$$k_B T = \frac{\hbar c^3}{Gm}. \tag{19}$$

Equation (19) is the well known Beckenstein temperature formula valid for a Black Hole of arbitrary mass but derived here for the Planck mass.

Can we now generalize (19) to the case of a Black Hole of arbitrary mass, as in the original Beckenstein formula but using only the characterization of the Black Hole in terms of Planck oscillators, as above? This is what we will do. In fact to a Black Hole of mass $M$ characterized in terms of $\bar{N}$ oscillators as in (15), we associate a Black Hole temperature defined by

$$\bar{T} = \frac{T}{\sqrt{\bar{N}}},$$

where $T$ is given in (19). ($\bar{N}$ here is not the number of particles in the universe). Using this with (15) in (19) we immediately get

$$k_B \bar{T} = \frac{\hbar c^3}{GM}. \tag{20}$$

Equation (20) which is the analogue of (19) is the required result. After this identification, we next use the following known relations for a Schwarzchild Black Hole [35]:

$$dM = TdS, \quad S = \frac{kc}{4\hbar G}A, \tag{21}$$

where $T$ is the Black Hole temperature, now identified with (20), $S$ the entropy and $A$ is the area of the Black Hole. The area is given by, using (15)

$$A = \bar{N} l^2 \tag{22}$$

because, this area is $\sim R^2$. Alternatively this shows that there are $\bar{N}$ elementary areas $l^2$ forming the Black Hole. Indeed this defines the basic quantum of area of quantum gravity approaches and is in pleasing agreement with the result of Baez deduced from a different quantum gravity consideration [36].

Using (15), (19) and (22), we can easily see that (21) is valid for the mass $M$ given by (15) or (14).

This completes the identification of Black Holes characterized by coherent Planck oscillators, with the conventional Hawking-Beckenstein theory.

## 7 Remark

As already noted, in one sense, we can get lengths $< l_P$ if the mass $< m_P$ though such a scale would no longer be in terms of the fundamental constants, unlike the Planck scale (1). However, let us consider the following relation (cf. Ref. [22]),

$$\omega_{max}^2 = \frac{c^2}{l^2}. \tag{23}$$

This follows from the theory of phonons in an array of coherent oscillators e.g. atoms in a linear crystal as in our model. If we use (23) in (16), then we get the Planck length (1). In other words, the Planck length is the result of not just a single oscillator but rather a whole array of oscillators as in our theory. A small scale would lead to an unphysical chaotic universe.

It is no longer arbitrarily prescribed as in (1). From this point of view, there is a distinction in the interpretation of gravitation as compared to Sakharov's formulation alluded to. True gravitation shows up as a residual energy according to (17) as in Sakharov's theory. But now, this is due to the result of the array of oscillators at the Planck scale in the background dark energy.

A further interesting remark is the following. When we consider an array of oscillators in considerations from (9) ff, it is worth remembering that these oscillators are actually coupled, due to the noncommutativity that is implicit as noted in Sect. 2 (cf. Ref. [37, 38] for further details).

## 8 Conclusion

We have shown that it is possible to consider the universe to have an underpinning of Planck oscillators in the background dark energy. This leads to a meaningful description of the universe of elementary particles and also of black hole thermodynamics. Finally it provides a description of gravitation, not as a separate fundamental interaction, but rather as the residual energy of the background dark energy that is a result of the fact that there is a minimum fundamental spacetime interval that is required for a meaningful universe.

## References


1. Kiefer, C.: Quantum Gravity. Clarendon, Oxford (2004)
2. Regge, T., de Alfaro, V.: Potential Scattering. North Holland, Amsterdam (1965)
3. Roman, P.: Advanced Quantum Theory, p. 31. Addison-Wesley, Reading (1965)
4. Bogdan, P., Rith, K.: Particles and Nuclei: An Introduction to the Physical Concepts. Springer, Berlin (1993)
5. Veneziano, G.: In: Huggett, S.A., et al. (eds.) The Geometric Universe. Oxford University Press, Oxford (1998)
6. Veneziano, G.: Phys. Rep. **9**(4), 199–242 (1974)
7. Lee, T.D.: Particle Physics and Introduction to Field Theory. Harwood Academic, Reading (1981)
8. Martin, B.R., Shaw, G.: Particle Physics. Wiley, New York (1992)
9. Sidharth, B.G.: Fuzzy, non commutative spacetime: a new paradigm for a new century. In: Sidharth, B.G., Altaisky, M.V. (eds.) Proceedings of Fourth International Symposium on "Frontiers of Fundamental Physics, pp. 97–108. Kluwer Academic/Plenum, New York (2001)
10. Fogleman, G.: Am. J. Phys. **55**(4), 330–336 (1987)
11. Schwarz, J.: Physics Reports **89**(227) (1982)
12. Jacob, M. (ed.) Physics Reports. North-Holland, Amsterdam (1974) reprint volume
13. Veneziano, G.: Quantum geometric origin of all forces in string theory. In: Huggett, S.A., et al. (eds.) The Geometric Universe, pp. 235–243. Oxford University Press, Oxford (1988)
14. Ramond, P.: Phys. Rev. D **3**(10), 2415–2418 (1971)
15. Kaluza, Th.: An Introduction to Kaluza-Klein Theories. Singapore, World Scientific (1984)
16. Greene, B.: The Elegant Universe, p. 15. Vintage, London (1999)
17. Madore, J.: An Introduction to Non-Commutative Differential Geometry. Cambridge University Press, Cambridge (1995)
18. Laughlin, R.B.: A Different Universe. Basic Books, New York (2005)
19. Smolin, L.: The Trouble with Physics. Houghton Mifflin Company, New York (2006)
20. Sidharth, B.G.: When the Universe Took a U Turn. Singapore, World Scientific (2009)
21. Susskind, L.: The Black Hole War. Back Bay Books, New York (2008)
22. Sidharth, B.G.: Thermodynamic Universe. Singapore, World Scientific (2008)
23. Sidharth, B.G.: Chaotic Universe: From the Planck to the Hubble Scale. Nova Science, New York (2001)
24. Sidharth, B.G.: The Universe of Fluctuations. Springer, Netherlands (2005)
25. Sidharth, B.G.: Found. Phys. Lett. **17**(5), 503–506 (2004)
26. Sidharth, B.G.: Found. Phys. Lett. **15**(6), 577–583 (2002)
27. Goodstein, D.L.: States of Matter. Dover, New York (1975)
28. Jack Ng, Y., Van Dam, H.: Mod. Phys. Lett. A **9**(4), 335–340 (1994)
29. Rosen, N.: Int. J. Theor. Phys. **32**(8), 1435–1440 (1993)
30. Sidharth, B.G.: Int. J. Mod. Phys. A **21**(31), 6315 (2006)
31. Sidharth, B.G.: Int. J. Theor. Phys. **48**(12), 3421 (2009)
32. Cercignani, C.: Found. Phys. Lett. **11**(2), 189–199 (1998)
33. Sakharov, A.D.: Sov. Phys. Dokl. **12**(11), 1040–1041 (1968)
34. Sidharth, B.G.: Int. J. Theor. Phys. **48**(8), 2427–2431 (2009)
35. Ruffini, R., Zang, L.Z.: Basic Concepts in Relativistic Astrophysics. Singapore, World Scientific (1983)
36. Baez, J.: Nature **421**, 702–703 (2003)
37. Sidharth, B.G.: In: Hartnett, J., et al. (eds.) Alternative Routes to Gravitation in Frontiers of Fundamental Physics. AIP, New York (2010)
38. Sidharth, B.G.: Energy and mass generation. arXiv:1003.2330